What is Statistical PERT®?

Statistical PERT® is a technique for making probabilistic estimates for bell-shaped uncertainties. A bell-shaped uncertainty – also referred to as the normal probability distribution, the Gaussian distribution, or, more-commonly, just the "the normal curve" or "the bell curve" – describes the shape of an uncertainty when it is graphically charted, where possible outcomes for the uncertainty are plotted on the x-axis, and the frequency of those possible outcomes is plotted on the y-axis.

A bell-shaped uncertainty is one that has a probable, most likely outcome (the statistical mode), plus highly improbable minimum and maximum outcomes. There are many bell-shaped uncertainties in life, such as student test scores for an exam given by a teacher, where the probable, most likely outcome might be a student scoring 70%, and only a very few students will improbably score either 100% or 40%. In business, revenue and expense forecasting deals with bell-shaped uncertainties, and many projects have bell-shaped uncertainty with respect to how long they take to complete, or how much work they will complete in a specific timeframe.

Statistical PERT improves upon PERT estimation (described below) by making use of Microsoft Excel®'s built-in statistical functions for the normal probability distribution. Using Microsoft Excel, Statistical PERT can create an infinite number of estimates for an uncertainty, with each estimate having its own, cumulative probability that it will (or will not be) exceeded by the actual outcome for that uncertainty.

What is PERT?

In the 1950s, the United States Navy developed the Program Evaluation and Review Technique[1] (PERT) and used it to improve scheduling large projects. Modern project management has used PERT as the basis for making estimates for project tasks or costs. PERT estimation relies upon the **PERT formula**, expressed in slightly different ways, but all having the same result of creating an arithmetic average (the mean) for a bell-shaped uncertainty. The PERT formula is:

$$PERT = \frac{(Optimistic + 4(Most\ Likely) + Pessimistic)}{6}$$

The PERT formula above creates an estimate of a PERT probability distribution[2], which is similar to the normal distribution, but all the points of a PERT distribution lie between the minimum (optimistic) and maximum (pessimistic) values. The normal distribution can also use the PERT formula to calculate the mean for a normal curve, but in plotting a normal curve, there are points on the curve that are smaller the minimum value used in the PERT formula, and there

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[1] https://en.wikipedia.org/wiki/Program_evaluation_and_review_technique
[2] The PERT probability distribution is a special form of the beta probability distribution which can assume many different shapes, including a bell-shaped appearance that may or may not be skewed. References to the PERT distribution in this white paper refer specifically to the PERT distribution created using Palisade's RiskPERT function in the @Risk Excel add-in program.

are points that exceed PERT's maximum value because of the characteristics of the normal distribution. The probabilities associated with such points, however, are very, very small – small enough that relying on the PERT formula to calculate an estimate of a normal curve mean usually poses little or no problem for an estimator.

Another, lesser-known formula is sometimes referred to as the **PERT standard deviation formula**[3]. This formula calculates a standard deviation for a bell-shaped uncertainty. The formula is expressed as:

$$PERT\ Standard\ Deviation = \frac{(Pessimistic - Optimistic)}{6}$$

By knowing these two formulas, an estimator can quickly choose an estimate using one, two or three standard deviations to create an estimate interval that has, respectively, 68%, 95% or 99.7% probability of occurrence. This makes use of the 68-95-99.7 Rule in statistics.

PERT in Project Management

In project management, the estimator is not usually concerned with the left-side tail of the bell-shaped curve when estimating task duration (that is, finishing a task extremely early), rather, the estimator is concerned with finishing a task late or very late, which is the right-side tail of the bell-shaped curve. Since most projects are scheduled using deterministic estimates for the completion of each task in the project, an estimator who wishes to use PERT estimation chooses a value that is to the right of the mode and mean, and often very close to the right-side tail of the probability curve. The probability associated with the left-side tail is added to the cumulative probability of the selected estimation value.

Example: Suppose a PERT estimate for some bell-shaped uncertainty had a mean of 100 days and a standard deviation of 10 days. If you added and subtracted two standard deviations from the mean, you would create an interval of 80 to 120 days, and 95% of all points on the bell-shaped curve would lie between those two points.[4] If an estimator did not care about the left-side tail, that area – which is one-half of the 5% not contained in the interval – could be added to the interval's 95% probability, so only the right-side tail is excluded. Therefore, an estimator could say with 97.5% probability that the bell-shaped uncertainty will be equal to or less than 120; the remaining 2.5% represents the area only in the right-side tail of the curve[5].

Using just PERT formulas alone, however, relegate an estimator to adding and subtracting one, two or three standard deviations to and from the mean. There is no easy way for an estimator to find the cumulative probability for other points on the bell-shaped curve without a computer.

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[3] https://en.wikipedia.org/wiki/Three-point_estimation
[4] The actual probability is ~95.45%
[5] 97.5% is an approximation only. A more precise percentage would be 97.725%.

This is one of the key obstacles that Statistical PERT overcomes. By relying on Excel's NORM.DIST and NORM.INV functions for the normal curve, Statistical PERT can very easily find the cumulative probability for **any** point on the normal curve. Using Statistical PERT, an estimator can start with a planning value to find its cumulative probability ("What is the likelihood my expense forecast of $100,000 will be exceeded?"), or, an estimator can specify a probability ("I need an estimate that has only a 15% probability of being exceeded.") and find its corresponding point on the normal curve.

What Else is Different about Statistical PERT?

A bell-shaped curve's specific shape is determined by its standard deviation (how spread out is the normal curve), its kurtosis (how much vertical rise does the normal curve have), and its skewness (how symmetrical is the normal curve).

Microsoft Excel's two built-in functions for the normal curve (NORM.DIST and NORM.INV) allow only a few arguments, but neither function allows for directly specifying the curve's kurtosis or skewness. Both functions, however, require the standard deviation to be specified, and the size of the standard deviation will influence the shape of a bell-shaped curve relative to other bell-shaped curves (even when all curves have the same mean).

The PERT standard deviation formula yields only one result for any given 3-point estimate. With Statistical PERT, an estimator can use their subjective opinion (intuition, gut instinct, private knowledge, etc.) about *how likely* the most likely outcome really is, relative to both the minimum and maximum point-estimates. Statistical PERT, then, has the ability to adjust probabilities around the mean in a way that PERT estimation cannot. By not being limited to just one standard deviation for a 3-point estimate, Statistical PERT lets estimators make qualitative judgements that create probabilistic estimates which reflect the estimator's subjective judgment.

How does Statistical PERT work?

Statistical PERT still uses a 3-point estimate for a bell-shaped uncertainty, and Statistical PERT also uses the PERT formula to estimate the mean of the normal curve. What's different about Statistical PERT is it replaces the PERT standard deviation formula with this simple formula:

$$(Pessimistic - Optimistic) \times Ratio\ Scale\ Multiplier$$

In this formula, the Ratio Scale Multiplier is a value between 0 and 1 that represents a subjective word phrase an estimator uses to describe *how likely* the most likely outcome really is. To make this very simple, Statistical PERT uses the **SPERT-7 Rule™** to equate multiplier values to six subjective opinions about *how likely* the most likely outcome really is. These are:

7%   Near Certainty
14%   High Confidence
21%   Medium-high Confidence

28%   Medium-low Confidence

35%   Low Confidence

42%   Guesstimate

Obviously, each of the Ratio Scale Multipliers is an increment of 7%. The Medium-high Confidence multiplier (21%) closely aligns with a Monte Carlo simulation of a PERT distribution using the same 3-point estimate[6]. The SPERT-7 Rule makes it easy to remember which multiplier to use for a particular subjective opinion, but Statistical PERT can work with less than or more than six subjective opinions, and the Statistical PERT template provides a way to calibrate the Ratio Scale Multipliers to more closely fit the characteristics of a particular uncertainty.

Scenario: Traveling from your home to arrive at the airport gate just before the first boarding announcement is given is a skewed, bell-shaped uncertainty. This uncertainty includes driving to the airport, parking, checking-in luggage, going through security, and walking to the gate. Suppose an estimator believes that the most likely outcome is that this will take a total of 120 minutes. At a bare minimum, under ideal conditions, this would take 60 minutes. In a worst-case scenario, it should take no longer than 240 minutes. Therefore, the 3-point estimate for traveling to the airport and arriving at the airport gate just as the first boarding call is made is 60-120-240 (minimum, most likely, maximum).

Using PERT estimation, the PERT formula creates the mean for a PERT distribution (which is also the used to create the mean for a normally-distributed probability):

$$\frac{(60 + 4(120) + 240}{6} = 130 \; minutes$$

Using Statistical PERT and the SPERT-7 Rule, suppose an estimator wants to obtain probabilities that closely track to a Monte Carlo simulation (that uses a PERT distribution) for this trip to the airport. Therefore, the estimator would have Medium-high Confidence in how likely the most likely outcome really is, and the SPERT-7 Rule dictates that the corresponding Ratio Scale Multiplier is 21%. To calculate the standard deviation for this travel-related uncertainty, the formula would be:

$$(240 - 60) \times 0.21 = 37.8 \; minutes$$

Using Excel's built-in statistical functions, Statistical PERT now has enough information to create an infinite number of probabilistic estimates. The two Excel functions required are (in Microsoft Excel 2010, Excel 2013 and Excel 2016) NORM.DIST and NORM.INV.

Suppose an estimator wants to be 95% confident in getting to the airport gate on-time. How many minutes will she need to plan for that trip? The Excel formula would be:

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[6] Using Palisade's @Risk Excel add-in program and the RiskPERT function, which creates random samples using a special form of the beta distribution

$$NORM.INV(0.95, 130, 37.8) = 192\ minutes$$

The estimator calculates that she will get to the airport on-time or early 95% of the time provided she leaves her home 192 minutes before the first boarding call is given. Although she expects that it will most likely take her 120 minutes, she will need to include a 72 minute buffer (120 + 72 = 192) to be 95% confident she will be on-time or early.

How closely does Statistical PERT's estimate of 192 minutes match an estimate obtained by a Monte Carlo simulation and the PERT distribution? Using Palisade's @Risk® software, the RiskPERT(60, 120, 240) function creates a standard deviation of 33 minutes and a 95% estimate of 188 minutes. The difference between a Statistical PERT estimate and a sophisticated, Monte Carlo simulation is four minutes.

What if the estimator has *high confidence* in getting to the airport in 120 minutes? In that case, Statistical PERT lets the estimator adjust the standard deviation by employing a different Ratio Scale Multiplier ("High Confidence") to match her subjective opinion about how likely the most likely outcome really is. High confidence equates to a 14% Ratio Scale Multiplier, so the Statistical PERT calculation for the standard deviation under high confidence is:

$$(240 - 60) \times 0.14 = 25.2\ minutes$$

And the Statistical PERT estimate at 95% confidence using this new standard deviation is:

$$NORM.INV(0.95, 130, 25.2) = 171\ minutes$$

When the estimator has high confidence in getting to the airport gate in the most likely outcome of 120 minutes, she needs to plan only 51 minutes of travel buffer time.

What happens if the estimator has *more* uncertainty – not *less* – around her most likely outcome? Perhaps she is traveling during rush hour, and travel times are much more unpredictable when there is heavy traffic. She still believes that the most likely outcome will take 120 minutes to travel to the airport gate, but now, she feels much more uneasy about the most likely result.

To create an estimate under greater uncertainty about how likely the most likely outcome really is, the estimator chooses a different subjective opinion – either Medium-Low, Low, or Guesstimate. Suppose she chooses Medium-Low, and the SPERT-7 Rule says that Medium-Low Confidence in the most likely outcome corresponds to a Ratio Scale Multiplier of 28%. Here is the revised Statistical PERT standard deviation for her trip:

$$(240 - 60) \times 0.28 = 50.4\ minutes$$

And the Statistical PERT estimate at 95% confidence using this revised standard deviation is:

$$NORM.INV(0.95, 130, 50.4) = 213\ minutes$$

With greater uncertainty around the mean of the normal curve, the estimator now needs 93 minutes of buffer time (213 – 120 = 93) to obtain 95% confidence she will arrive to the airport on-time or early.

Using Statistical PERT, the estimator uses the same 3-point estimate to make three different estimates, all of which have 95% confidence: 171, 192 and 213 minutes. Each estimate is tied to the estimator's different, subjective opinion about how likely the most likely outcome really is. Statistical PERT allows an estimator to make a subjective opinion about a bell-shaped uncertainty, and that subjective opinion influences the shape of the curve and the resulting probabilities of different points on the curve.

Statistical PERT can also use the NORM.DIST function, too, to find the cumulative probability of a planning estimate. Suppose the estimator wanted to know what her probability of success would be in getting to the airport on-time or early if she left her house 150 minutes before the first boarding call would be given for her flight; this would give her only a 30 minute buffer. Under a condition of Medium-High confidence in the most likely outcome, the Statistical PERT standard deviation would still be 37.8 minutes, so the calculation using the NORM.DIST function would look like this:

$$NORM.DIST(150, 130, 37.8, TRUE) = 70\%$$

If the estimator only adds 30 minutes to her most likely outcome of 120 minutes, she will get to the airport on-time or early only 70% of the time. She estimates that 30% of the time, her travel time will take longer than 150 minutes, and so she risks missing her flight nearly one out of three attempts if she allows for only a 30 minute buffer.

Calibrating Statistical PERT

The SPERT-7 Rule makes it easy to equate six different Ratio Scale Multipliers with six different, subjective opinions about how likely the most likely outcome really is. But Statistical PERT can be calibrated to create more-accurate ratio scale values. Using the Ratio Scale Modeler worksheet included on every Statistical PERT Excel download, an estimator can model uncertainties that match what exactly the estimator is estimating. Using the Ratio Scale Modeler, an estimator can distribute 100 hypothetical trials using only the 3-point estimate values; the modeler will calculate the resulting ratio scale multiplier for that distribution.

Calibration for most likely outcomes that are nearly certain to occur usually use a 1-98-1 split (minimum, most likely, maximum) of the 100 hypothetical trials. The resulting ratio scale multipliers are generally between 7% and 8%. Under conditions of great uncertainty surrounding the most likely outcome, the split is 33-34-33 for the 100 hypothetical trials. This leads to ratio scale multipliers that are generally around 40% to 43%. Between these two extremes, estimators can create an unlimited number of subjective opinions and choose any reasonable way to create intervals between those opinions.

Statistical PERT downloads all come pre-calibrated with **seven** subjective opinions (they include all the same opinions as the SPERT-7 Rule, but don't use the SPERT-7 Rule Ratio Scale Multiplier values; they also add a "Medium Confidence" opinion, too). For most bell-shaped uncertainties, the "Medium Confidence" opinion corresponds very closely to a Monte Carlo simulation using the PERT distribution[7]. Estimators can add to, subtract from, or modify these opinions and their corresponding ratio scale multipliers. They are all contained in the VLOOKUPS worksheet of the Statistical PERT download file.

Where Can Statistical PERT be used?

Statistical PERT can be used anytime you are faced with an uncertainty with bell-shaped properties and you can reasonably estimate the minimum, most likely, and maximum values for that uncertainty. And with Statistical PERT, you express a subjective opinion about *how likely* the most likely outcome really is, and the resulting probabilistic estimates will conform to your subjective opinion about the uncertainty.

Examples of uncertainties which can use Statistical PERT:

- Project or task duration or work effort
- Expenses
- Revenues
- Start dates or finish dates
- Project portfolio cost
- Product launch dates, release dates
- Event attendance, ticket sales
- Quality control measurements

Conclusion

Statistical PERT is an easy-to-use technique for creating probabilistic estimates for any bell-shaped uncertainty. Statistical PERT improves upon PERT estimation by allowing the estimator to choose from an infinite number of estimates, each having its own cumulative probability for success (or failure). Moreover, Statistical PERT allows the estimator to manipulate the standard deviation for the probability curve by using the SPERT-7 Rule and Ratio Scale Multipliers that correspond to the estimator's subjective opinion about *how likely* the most likely outcome really is. By manipulating the standard deviation, the estimator creates probabilistic estimates that better align with the estimator's expert knowledge about the uncertainty.

Statistical PERT Excel templates and example workbooks are freely available. You may modify and redistribute the files under the GNU General Public License[8]. To download Statistical PERT, go to http://www.statisticalpert.com.

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[7] Using Palisade's @Risk Excel add-in program and the RiskPERT function
[8] http://www.gnu.org/licenses/gpl.html